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Passive Decomposition of Mechanical Systems With Coordination Requirement

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***Abstract*—We show the fundamental *passive decomposition* property of general mechanical systems on a n-dim. configuration manifold $\mathcal{M}$, i.e., when endowed with a submersion $h : \mathcal{M} \to \mathcal{N}$, where $\mathcal{N}$ is a m-dim. manifold $(m \leq n)$, their Lagrangian dynamics with the kinetic energy as the Lagrangian can always be decomposed into: 1) shape system, describing the m-dim. dynamics of $h(q)$ on $\mathcal{N}$; 2) locked system, representing the $(n - m)$-dim. dynamics along the level set of h; and 3) energetically-conservative coupling between them. The locked and shape systems also individually inherit the Lagrangian structure and passivity of the original dynamics. We exhibit and analyze geometric and energetic properties of the passive decomposition in a coordinate-free manner. An illustrative example on SO(3) is also provided.**

***Index Terms*—Coordination, decomposition, differential geometry, Lagrangian systems, passivity.**

I. Introduction

Consider a mechanical system, evolving on a n-dim. configuration manifold $\mathcal{M}$ with its kinetic energy as the Lagrangian and endowed with a smooth submersion $h : \mathcal{M} \to \mathcal{N}$ with $h(q) \in \mathcal{N}$ specifying a certain coordination aspect (e.g., internal posture or grasping shape), where $q \in \mathcal{M}$ is the system's configuration and $\mathcal{N}$ is a m-dim. manifold with $m \leq n$. In this technical note, we show a fundamental property of the mechanical system in this setting, i.e., its n-dim. Lagrangian dynamics on $\mathcal{M}$ can be decomposed into: 1) shape system, describing the m-dim. dynamics of the coordination aspect $h(q)$ on $\mathcal{N}$ (e.g., grasping shape); 2) locked system,[1] representing the system's $(n-m)$-dynamics along the level set of h (e.g., motion of the grasped shape); and 3) inertia-induced energetically conservative coupling between them, which is a function of $(q, \dot{q})$ and quadratic in $\dot{q}$. The (decoupled) locked and shape systems also individually inherit the Lagrangian structure and passivity of the original dynamics. Due to these preserved Lagrangian structure and passivity, we call the decomposition *passive decomposition*.

This passive decomposition then allows us to achieve: 1) simultaneous *and* separate locked-shape control, which is necessary, e.g., for the precise multirobot grasping, where the grasping shape $h(q)$ (i.e., shape system) and the grasped object's behavior (i.e., locked system) should be controlled together with no crosstalk between them; and 2) exploitation of the (preserved) Lagrangian structure and passivity for the locked and shape control synthesis (e.g., passivity-based control; stability via passivity [3]). Due to these practically-useful properties, passive decomposition has been applied to various applications [3]–[6]. However, these prior results are limited to $\mathcal{M} = \Re^n$ and $\mathcal{N} = \Re^m$ (thus, inapplicable, e.g., to SO(3)—see Section IV) and some fundamental geometry-related questions are not answered there (e.g., why the shape system is representable by a Lagrangian-like dynamics on $\mathcal{N}$; why the locked-shape configuration decomposition is generally impossible, etc.). In this technical note, we present passive decomposition on a manifold $\mathcal{M}$ in a coordinate-free manner, and delineate its important geometric and energetic properties. A portion of this technical note was presented in [7] and [8].

Some relevant results in the literature and their comparison with our results in this technical note are as follows: 1) constrained dynamics approach [9], [10], which assumes $h(q) = c$, thus, is not suitable when $h(q)$ needs to be controlled (e.g., squeezing grasping); 2) feedback linearization [11], [12], which typically aims to eliminate the underlying Lagrangian dynamics and passivity, rather than exploit them; and 3) impedance control [13], in which the locked-shape coupling is usually left uncompensated for; 4) power-continuous decomposition of [14], which is limited only to the curve-tracking (i.e., $m = n - 1$) and $\Re^n$-coordinates; and 5) Lagrangian reduction [1], [2], from which we adopt the terms, "shape" and "locked", yet, symmetry is required and passivity overlooked there.

The rest of the technical note is organized as follows. Section II introduces some preliminary materials. Geometric and energetic properties of the passive decomposition are detailed in Section III. An illustrative example is given in Section IV. Section V concludes the technical note.

II. Preliminary

A. Geometry of Mechanical Systems

We consider a mechanical system, whose configuration q evolves on a n-dim. smooth manifold $\mathcal{M}$ with the velocity $v := \dot{q} \in T_q\mathcal{M}$ and the external/control force $T, F \in T_q^*\mathcal{M}$, where $T_q\mathcal{M}$ and $T_q^*\mathcal{M}$ are respectively the tangent and cotangent spaces at $q \in \mathcal{M}$. We denote its (differentiable Riemannian) inertia metric by M [15], which assigns, for each $q \in \mathcal{M}$, an inner product $\langle\langle \ \rangle\rangle$ on $T_q\mathcal{M}$, defines the system's kinetic energy s.t.

$$\kappa(t) := \frac{1}{2} \langle\langle \dot{q}(t), \dot{q}(t) \rangle\rangle \quad (1)$$

[1]We may view the coordination aspect $h(q)$ as "output" and the locked system as "internal dynamics". This viewpoint, however, we do not pursue here, since: 1) $h(q)$ specifically describes (configuration) coordination aspect among q; 2) *equally*-rich controlled behaviors of the locked and shape systems are often desired/attainable; and 3) our passive decomposition is influenced by the locked-shape concepts of [1], [2].

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and also begets a linear isomorphism $M(q) : T_q\mathcal{M} \to T_q^*\mathcal{M}$ as defined by

$$\langle M(q)v_1, v_2\rangle := \langle\langle v_1, v_2\rangle\rangle, \qquad v_1, v_2 \in T_q\mathcal{M} \tag{2}$$

where $\langle\ \rangle : T_q^*\mathcal{M} \times T_q\mathcal{M} \to \Re$ is the standard pairing. We also assume that $\mathcal{M}$ is paracompact and second-countable [16].

The dynamics of the mechanical system on $\mathcal{M}$ is then given by the Lagrange equation[2]

$$M(q)\nabla_{\dot q} v = T + F \tag{3}$$

where ∇ is the Levi-Civita connection on $\mathcal{M}$ [15] with the following properties: i) it is affine

$$\begin{aligned}\nabla_{fX+gY} Z &= f\nabla_X Z + g\nabla_Y Z\\ \nabla_X(Y+Z) &= \nabla_X Y + \nabla_X Z\\ \nabla_X(fY) &= f\nabla_X Y + \mathcal{L}_X(f)Y\end{aligned} \tag{4}$$

where the last equality is called Leibniz property of ∇_X; ii) it is compatible w.r.t. the M-metric

$$\mathcal{L}_X\langle\langle Y, Z\rangle\rangle = \langle\langle \nabla_X Y, Z\rangle\rangle + \langle\langle Y, \nabla_X Z\rangle\rangle \tag{5}$$

and, iii) it is torsion-free

$$T(X,Y) := \nabla_X Y - \nabla_Y X - [X,Y] = 0 \tag{6}$$

for all $X, Y, Z \in \mathfrak{X}(\mathcal{M})$ and $f, g \in C^\infty(\mathcal{M})$, where $C^\infty(\mathcal{M})$ is the set of all real smooth functions; $\mathfrak{X}(\mathcal{M})$ (or, $\mathfrak{X}^*(\mathcal{M})$, resp.) is that of all smooth vector (or, covector, resp.) fields on $\mathcal{M}$; $\mathcal{L}_X f$ is the Lie derivative of f along X; and [,] is the Lie bracket, defined by $\mathcal{L}_{[X,Y]}f = \mathcal{L}_X\mathcal{L}_Y f - \mathcal{L}_Y\mathcal{L}_X f$. In (3), we assume that all the external forces (e.g., gravity) are embedded in F. This equation (3) can also capture multiple mechanical systems, when formulated on their product manifold [19].

In this technical note, we assume that a certain coordination aspect of (3) (e.g., internal posture or grasping shape) can be described by the image $h(q)$ of a smooth map

$$h : \mathcal{M} \to \mathcal{N}, \qquad n \geq m \tag{7}$$

where $\mathcal{N}$ is a m-dim. smooth manifold. We also assume that h is a submersion [15], i.e., its push-forward[3] $h_* : T_q\mathcal{M} \to T_{h(q)}\mathcal{N}$ is surjective $\forall q \in \mathcal{M}$. Then, each level set

$$\mathcal{H}_{h(q)} := \{p \in \mathcal{M} | h(p) = h(q)\} \tag{8}$$

defines a $(n-m)$-dim. submanifold in $\mathcal{M}$, and their collection forms a foliation [20]. See Fig. 1. We will call h *coordination map* and $\mathcal{N}$ *coordination manifold*.

From the compatibility (5), we have $d\kappa(t)/dt = \langle M\nabla_{\dot q} v, v\rangle = \langle F + T, v\rangle$. Integrating this, we can then show the passivity of (3): for all $\bar T \geq 0$

$$\int_0^{\bar T} \langle T + F, v\rangle dt = \kappa(\bar T) - \kappa(0) \geq -\kappa(0). \tag{9}$$

Our passive decomposition aims to decompose the dynamics (3) according to the coordination map h and the M-metric, while preserving the Lagrangian structure and passivity of (3), which are often useful for control synthesis (e.g., passivity-based control [21]; stability via passivity [3]).

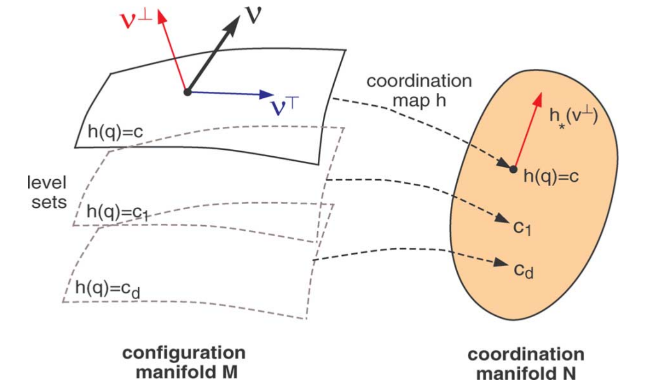

Fig. 1. Geometry of passive decomposition.

III. Passive Decomposition

A. Tangent and Cotangent Space Decomposition

Given the coordination map h and the M-metric, we can decompose the tangent space $T_q\mathcal{M}$ of (3) s.t., at each $q \in \mathcal{M}$

$$T_q\mathcal{M} = T_q^\top\mathcal{M} \oplus T_q^\perp\mathcal{M} \tag{10}$$

where[3]

$$\begin{aligned} T_q^\top\mathcal{M} &:= \ker(h_*) = \text{span}\{v \in T_q\mathcal{M} | h_*(v) = 0\}\\ T_q^\perp\mathcal{M} &:= \text{span}\left\{v' \in T_q\mathcal{M} | \langle\langle v, v'\rangle\rangle = 0, \quad \forall v \in T_q^\top\mathcal{M}\right\}.\end{aligned}$$

Collecting $T_q^\top\mathcal{M}$ and $T_q^\perp\mathcal{M}$ over $q \in \mathcal{M}$, we can then construct the tangential and normal distributions, $\Delta^\top$ and $\Delta^\perp$, s.t. $\Delta^\top(q) := T_q^\top\mathcal{M}$ and $\Delta^\perp(q) := T_q^\perp\mathcal{M}$. Here, $\Delta^\top$ and $\Delta^\perp$ are both regular with $\dim(\Delta^\top) = n - m$ and $\dim(\Delta^\top) = m, \forall q \in \mathcal{M}$. Also, $\Delta^\top$ is integrable with $\mathcal{H}_{h(q)}$ as its integral manifold; $\Delta^\perp$ is generally not.

With the tangent space decomposition (10), the cotangent space $T_q^*\mathcal{M}$ also splits s.t.

$$T_q^*\mathcal{M} = T_q^{*\top}\mathcal{M} \oplus T_q^{*\perp}\mathcal{M} \tag{11}$$

where

$$\begin{aligned} T_q^{*\top}\mathcal{M} &:= \text{span}\left\{w \in T_q^*\mathcal{M} | \langle w, v'\rangle = 0, \quad \forall v' \in T_q^\perp\mathcal{M}\right\}\\ T_q^{*\perp}\mathcal{M} &:= \text{span}\left\{w' \in T_q^*\mathcal{M} | \langle w', v\rangle = 0, \quad \forall v \in T_q^\top\mathcal{M}\right\}.\end{aligned}$$

Collecting $T_q^{*\top}\mathcal{M}$ and $T_q^{*\perp}\mathcal{M}$, we can similarly construct the tangential and normal codistributions, $\Omega^\top$ and $\Omega^\perp$, s.t. $\Omega^\top(q) := T_q^{*\top}\mathcal{M}$ and $\Omega^\perp(q) := T_q^{*\perp}\mathcal{M}$. Note that $\Omega^\perp$ and $\Omega^\top$ annihilate $\Delta^\top$ and $\Delta^\perp$, respectively. Using (10) and (11), we can also decompose $X \in \mathfrak{X}(\mathcal{M})$ (or $w \in \mathfrak{X}^*(\mathcal{M})$, resp.) s.t. $X = X^\top + X^\perp$ (or $w = w^\top + w^\perp$, resp.) with $X^\top \in \Delta^\top$, $X^\perp \in \Delta^\perp$ (or $w^\top \in \Omega^\top$, $w^\perp \in \Omega^\perp$, resp.).

B. Decomposition of Dynamics

Using (10), we decompose the velocity $v := \dot q \in T_q\mathcal{M}$ of (3) s.t.

$$v = v^\top + v^\perp \tag{12}$$

where we call 1) $v^\top \in T_q^\top\mathcal{M}$, *locked velocity*, since, being tangent to $\mathcal{H}_{h(q)}$, it will describe the motion of (3) when the coordination is locked (i.e., $dh/dt = 0$ with $v^\perp = 0$); and 2) $v^\perp \in T_q^\perp\mathcal{M}$, *shape*

[2]The term $\nabla_{\dot q} v$ of (3) should be understood as follows [15], [17]: for $w(t) \in T_{q(t)}\mathcal{M}$, $\nabla_{\dot q(t)} w(t) = (\nabla_X W)(q(t))$, where $X, W \in \mathfrak{X}(\mathcal{M})$ are (local) extension vector fields [18] to $\dot q(t)$ and $w(t)$ at $q(t)$ s.t. $X(q(t)) = \dot q(t)$ and $W(q(\tau)) = w(\tau), \forall \tau \in (t - \epsilon, t + \epsilon)$ for small $\epsilon > 0$.

[3]We use h_* to denote both $h_{*q} : T_q\mathcal{M} \to T_{h(q)}\mathcal{N}$ and $h_* : \mathfrak{X}(\mathcal{M}) \to \mathfrak{X}(\mathcal{N})$. Similar also holds for h^* and l_*.

velocity, which specifies how the coordination aspect $h(q)$ changes on $\mathcal{N}$ s.t.

$$\frac{d}{dt}h(q) = h_*(v^\top + v^\perp) = h_*(v^\perp) \in T_{h(q)}\mathcal{N}. \tag{13}$$

Note that the decomposition (12) is orthogonal and isometric, i.e., $\langle\langle v^\top, v^\perp\rangle\rangle = 0$ and $\langle\langle v, v\rangle\rangle = \langle\langle v^\top, v^\top\rangle\rangle + \langle\langle v^\perp, v^\perp\rangle\rangle$. Similarly, we can also decompose $F, T \in T_q^*\mathcal{M}$ s.t.

$$F = F^\top + F^\perp, \quad T = T^\top + T^\perp \tag{14}$$

where $\star^\top \in T_q^{*\top}\mathcal{M}$ and $\star^\perp \in T_q^{*\perp}\mathcal{M}$ respectively affect the motion of (3) along $\mathcal{H}_{h(q)}$ and the coordination aspect $h(q)$ on $\mathcal{N}$.

Using these, we can then decompose the Lagrangian dynamics (3) on $\mathcal{M}$ s.t.

$$\begin{aligned} M\nabla_{\dot q} v &= M(\nabla_{\dot q} v^\top)^\top + M(\nabla_{\dot q} v^\top)^\perp + M(\nabla_{\dot q} v^\perp)^\top \\ &\quad + M(\nabla_{\dot q} v^\perp)^\perp \\ &= T^\top + T^\perp + F^\top + F^\perp \end{aligned} \tag{15}$$

from which we can obtain[4]

$$M(\nabla_{\dot q} v^\top)^\top = -M(\nabla_{\dot q} v^\perp)^\top + T^\top + F^\top \tag{16}$$
$$M(\nabla_{\dot q} v^\perp)^\perp = -M(\nabla_{\dot q} v^\top)^\perp + T^\perp + F^\perp \tag{17}$$

where 1) we call $M(\nabla_{\dot q} v^\top)^\top$ in (16), *locked system dynamics*, which describes the $(n-m)$-dim. dynamics of (3) along the level set $\mathcal{H}_{h(q)}$—some of its geometric properties are detailed in Section III-E; 2) we call $M(\nabla_{\dot q} v^\perp)^\perp$ in (17), *shape system dynamics*, which specifies the m-dim. dynamics of $h(q)$ on $\mathcal{N}$, as formally elaborated in Section III-D; and 3) $M(\nabla_{\dot q} v^\top)^\perp$ and $M(\nabla_{\dot q} v^\perp)^\top$ define the locked-shape coupling, which is energetically conservative, a function of $(q, \dot q)$, and quadratic in $\dot q$, as shown in Section III-C.

C. Energetics of the Passive Decomposition

The decomposition (12) and (14) also decomposes the kinetic energy (1) and the power of (3) s.t.

$$\kappa(t) = \frac{1}{2}\langle\langle v^\top, v^\top\rangle\rangle + \frac{1}{2}\langle\langle v^\perp, v^\perp\rangle\rangle$$

and $\langle T, v\rangle = \langle T^\top, v^\top\rangle + \langle T^\perp, v^\perp\rangle$ (similar also holds for F). From (2), (5), (16), and (17), we also have

$$\frac{d}{dt}\kappa_l(t) = \langle T^\top, v^\top\rangle + \langle F^\top, v^\top\rangle - \left\langle M(\nabla_{\dot q} v^\perp)^\top, v^\top\right\rangle \tag{18}$$
$$\frac{d}{dt}\kappa_h(t) = \langle T^\top, v^\perp\rangle + \langle F^\perp, v^\perp\rangle - \left\langle M(\nabla_{\dot q} v^\top)^\perp, v^\perp\right\rangle \tag{19}$$

where $\kappa_l(t) := \langle\langle v^\top, v^\top\rangle\rangle/2$ and $\kappa_h(t) := \langle\langle v^\perp, v^\perp\rangle\rangle/2$ are the locked and shape kinetic energies. This shows that both the locked and shape systems have three power ports: control port, $\langle T^\top, v^\top\rangle$ and $\langle T^\perp, v^\perp\rangle$; external force port, $\langle F^\top, v^\top\rangle$ and $\langle F^\perp, v^\perp\rangle$; and locked-shape coupling port, $\langle M(\nabla_{\dot q} v^\perp)^\top, v^\top\rangle$ and $\langle M(\nabla_{\dot q} v^\top)^\perp, v^\perp\rangle$.

A remarkable property of our decomposition is that the locked-shape coupling is energetically conservative, that is

$$\begin{aligned} &\left\langle M(\nabla_{\dot q} v^\perp)^\top, v^\top\right\rangle + \left\langle M(\nabla_{\dot q} v^\top)^\perp, v^\perp\right\rangle \\ &= \langle\langle \nabla_{\dot q} v^\top, v^\perp\rangle\rangle + \langle\langle \nabla_{\dot q} v^\perp, v^\top\rangle\rangle \\ &= \mathcal{L}_{\dot q}\langle\langle v^\top, v^\perp\rangle\rangle = 0. \end{aligned} \tag{20}$$

Let us also write this coupling in coordinates. For this, define a basis set of $T_q\mathcal{M}$ by $\{\tilde e_1, \tilde e_2, \ldots, \tilde e_n\}$, s.t. $T_q^\top\mathcal{M} = \mathrm{span}\{\tilde e_1, \ldots, \tilde e_{n-m}\}$ and $T_q^\perp\mathcal{M} = \mathrm{span}\{\tilde e_{n-m+1}, \ldots, \tilde e_n\}$. Define also the Christoffel's symbols $\tilde\Gamma_{ij}^k(q)$ by $\nabla_{\partial/\partial q_i}\tilde e_j =: \sum_{k=1}^n \tilde\Gamma_{ij}^k(q)\tilde e_k$ [15]. Then, using $\dot q = \sum_{i=1}^n v_i(\partial/\partial q_i) = \sum_{i=1}^n \tilde v_i \tilde e_i$ and (4), we can write

$$(\nabla_{\dot q} v^\top)^\perp = \sum_{i=1}^n \sum_{j=1}^{n-m} \sum_{k=n-m+1}^n v_i \tilde v_j \tilde\Gamma_{ij}^k(q)\tilde e_k \tag{21}$$

and $(\nabla_{\dot q} v^\perp)^\top = \sum_{i=1}^n \sum_{j=n-m+1}^n \sum_{k=1}^{n-m} v_i \tilde v_j \tilde\Gamma_{ij}^k(q)\tilde e_k$. This shows that the locked-shape coupling is quadratic in $\dot q$, yet, still a function of $(q, \dot q)$, which are usually available in practice.

In many applications, this locked-shape coupling, $M(\nabla_{\dot q} v^\perp)^\top$ and $M(\nabla_{\dot q} v^\top)^\perp$, needs to be suppressed.[5] This is particularly so for faster operations, since it is quadratic in $\dot q$ (see (21)). For this, we design the decoupling control $T_d(q, \dot q)$ s.t.

$$T = \underbrace{M(\nabla_{\dot q} v^\top)^\perp + M(\nabla_{\dot q} v^\perp)^\top}_{=:T_d:\text{decoupling control}} + T_\star \tag{22}$$

where $T_\star$ is to embed additional control (Section III-F). Then, from (16), (17) and (18), (19) with this T_d, we can see that both the (decoupled) locked and shape systems will have the dynamics structure and passivity similar to (3). Moreover, this decoupling control T_d itself is passive[6] (i.e., $\langle T_d, v\rangle = 0$ from (20)), thus, consequently, the original system (3), when decoupled with (22), will still possess the same passivity (9) with T replaced by $T_\star$. Due to this preservation of the Lagrangian structure (3) and passivity (9) and this passive decoupling property, we name our decomposition *passive decomposition*.

D. Shape System on Coordination Manifold $\mathcal{N}$

It is often desired to put some priority on the task of controlling the coordination aspect $h(q)$ (e.g., maintaining grasping shape). In this Section III-D, we show that this coordination aspect $h(q)$ can be described on $\mathcal{N}$, so that we can design and analyze a control for it solely on the coordination manifold $\mathcal{N}$ with a lesser dimension $m \le n$.

For this, note that we already have the kinematics of $h(q)$ on $\mathcal{N}$, i.e., $dh(q)/dt = h_*(v^\perp) \in T_{h(q)}\mathcal{N}$. To describe the dynamics of $h(q)$ on $\mathcal{N}$, we define *shape system connection* $\nabla^h : \mathfrak{X}(\mathcal{M}) \times \mathfrak{X}(h(\mathcal{M})) \to \mathfrak{X}(h(\mathcal{M}))$ s.t.

$$\nabla_X^h Y^h := h_*(\nabla_X Y^\perp) \tag{23}$$

where $X \in \mathfrak{X}(\mathcal{M})$, $Y^h \in \mathfrak{X}(h(\mathcal{M}))$ and $Y^\perp \in \Delta^\perp$ with $Y^h = h_* Y^\perp$. Here, given $Y^h \in \mathfrak{X}(h(\mathcal{M}))$, $\exists$ an unique $Y^\perp \in \Delta^\perp$, since, being surjective, h_* defines a bijective map between $T_q^\perp\mathcal{M}$ and $T_{h(q)}\mathcal{N}$, $\forall q \in \mathcal{M}$ [8]. Recall also that submersions define open mappings [20]. Define also the induced M_h-metric on $\mathcal{N}$ by

$$\left\langle\left\langle v_1^h, v_2^h\right\rangle\right\rangle_{\mathcal{N}} := \left\langle\left\langle v_1^\perp, v_2^\perp\right\rangle\right\rangle_{\mathcal{M}} \tag{24}$$

for $v_1^h = h_* v_1^\perp$ and $v_2^h = h_* v_2^\perp$ with $v_1^\perp, v_2^\perp \in T_q^\perp\mathcal{M}$.

[4]As in the constrained dynamics approach [9], [10], if the coordination is locked with $h(q) = c$ and $v^\perp = 0$, $M(\nabla_{\dot q} v^\perp)^\top$ and $M(\nabla_{\dot q} v^\perp)^\perp$ vanish, and $M(\nabla_{\dot q} v^\top)^\top$ and $M(\nabla_{\dot q} v^\top)^\perp$ respectively reduce to the induced Levi-Civita dynamics on $\mathcal{H}_c$ and the second fundamental form [15, Ch.6]. In this technical note, we are yet interested in directly controlling $h(q)$ on $\mathcal{N}$.

[5]In the multirobot fixture-less grasping with h and $v^\top$ respectively describing the grasping shape and the grasped object's motion, with such a crosstalk, driving the grasped object via $v^\top$ can perturb the grasping shape $h(q)$ (e.g., dropping the object).

[6]This passivity of T_d is also robust, since, even with an incorrect estimate $\hat M(q)$, we still have $\langle \hat T_d, v\rangle = 0$ (i.e., (20) is invariant w.r.t. the choice of $\hat M(q)$), although, in this case, the locked-shape decoupling would not be perfect.

The shape system connection ∇^h (23) then allows us to map the shape system dynamics of (17) to $\mathcal{N}$ s.t.

$$\nabla^h_{\dot{q}} h_*(v^\perp) = h_* \left[(\nabla_{\dot{q}} v^\perp)^\perp \right]$$

which, with the kinematics relation $h_*(v^\perp) = dh(q)/dt$, now completes the (second-order dynamics) description of $h(q)$ on $\mathcal{N}$. The affine property and compatibility of ∇^h w.r.t. the M_h-metric, to be shown in the following Theorem 1, imply that this "mapped" shape system dynamics of $h(q)$ on $\mathcal{N}$ again possesses the Lagrangian structure and passivity.

Theorem 1: The shape system connection ∇^h (23) has the following properties: i) it is affine

$$\begin{aligned} \nabla^h_{fX+gX'} Z^h &= f\nabla^h_X Z^h + g\nabla^h_{X'} Z^h \\ \nabla^h_X (Y^h + Z^h) &= \nabla^h_X Y^h + \nabla^h_X Z^h \\ \nabla^h_X (fY^h) &= f\nabla^h_X Y^h + \mathcal{L}_X(f) Y^h \end{aligned} \tag{25}$$

ii) it is compatible w.r.t. the M_h-metric

$$\mathcal{L}_X \langle\langle Y^h, Z^h \rangle\rangle = \left\langle\left\langle \nabla^h_X Y^h, Z^h \right\rangle\right\rangle + \left\langle\left\langle Y^h, \nabla^h_X Z^h \right\rangle\right\rangle \tag{26}$$

for any $f, g \in C^\infty(\mathcal{M})$, $X, X' \in \mathfrak{X}(\mathcal{M})$, and $Y^h, Z^h \in \mathfrak{X}(h(\mathcal{M}))$.

Proof: The third item of (25) can be shown s.t.

$$\begin{aligned} \nabla^h_X (fY^h) &= h_*(\nabla_X f Y^\perp) = h_* \left(f\nabla_X Y^\perp + \mathcal{L}_X(f) Y^\perp \right) \\ &= f\nabla^h_X Y^h + \mathcal{L}_X(f) Y^h \end{aligned}$$

(other items can be proved similarly), while the compatibility (26) can be proved s.t., with (5)

$$\begin{aligned} \mathcal{L}_X \langle\langle Y^h, Z^h \rangle\rangle &= \mathcal{L}_X \langle\langle Y^\perp, Z^\perp \rangle\rangle \\ &= \left\langle\left\langle (\nabla_X Y^\perp)^\perp, Z^\perp \right\rangle\right\rangle + \left\langle\left\langle (\nabla_X Z^\perp)^\perp, Y^\perp \right\rangle\right\rangle \\ &= \left\langle\left\langle \nabla^h_z Y^h, Z^h \right\rangle\right\rangle + \left\langle\left\langle \nabla^h_X Z^h, Y^h \right\rangle\right\rangle \end{aligned}$$

where $Y^\perp, Z^\perp \in \Delta^\perp$ are the unique solutions of $Y^h = h_* Y^\perp, Z^h = h_* Z^\perp$, as stated after (23). ∎

The shape system connection ∇^h (23) can be thought of as the "projected connection" on $\Delta^\perp$ [22], transported by h_* to $\mathcal{N}$. This ∇^h may also be thought of as a connection over the map h [18]. In particular, when restricted on $\Delta^\perp$, ∇^h becomes the (Levi-Civita like) unique torsion-free and compatible connection over h w.r.t. the M_h-metric—see [8], [18].

E. Projection of Locked System

Suppose that there is a smooth submersion $l : \mathcal{M} \to \mathcal{L}$, where $\mathcal{L}$ is a $(n-m)$-dim. smooth manifold. Suppose also that, similar to (13), its push-forward $l_* : T_q\mathcal{M} \to T_{l(q)}\mathcal{L}$ satisfies the following *projectability* condition:

$$\frac{d}{dt} l(q) = l_*(v^\top + v^\perp) = l_*(v^\top) \in T_{l(q)}\mathcal{L} \tag{27}$$

$\forall q \in \mathcal{M}$ and $\forall v \in T_q\mathcal{M}$. If there exists such a projection pair $(l, \mathcal{L})$, we can then project the locked system of (16) to $\mathcal{L}$ as done for the shape system in Section III-D, and, consequently, the original system (3) can be passively *configuration-level* decomposed[7] into $\mathcal{L}$ and $\mathcal{N}$ (i.e., passive configuration decomposition [23]), on each of which we can control $l(q)$ and $h(q)$ individually. The next Proposition 1 shows that the integrability of $\Delta^\perp$ is necessary for such a pair $(l, \mathcal{L})$ to exist, which is in general not granted (e.g., Section IV). Note that, in contrast, $\Delta^\top$ is integrable from its construction (10).

Proposition 1: Suppose there exists a projection pair $(l, \mathcal{L})$ as defined above. Then, $\Delta^\perp$ is integrable.

Proof: Since l is a smooth submersion, we can find a m-dim. submanifold $\mathcal{G}_{l(q)} := \{p \in \mathcal{M} | l(p) = l(q)\} \forall q \in \mathcal{M}$. Also, from (27), we have $l_*(v^\perp) = 0, \forall v^\perp \in \Delta^\perp(q)$ and $\forall q \in \mathcal{M}$, implying that $\Delta^\perp \subset \ker(l_*)$. Now, suppose $\ker(l_*) \neq \Delta^\perp$. Then, from (10), there should exist a $v^\top \in \Delta^\top$ s.t. $l_*(v^\top) = 0$, which is yet impossible, since, similar to h_* (see the statement after (23)), l_* defines a bijective map between $\Delta^\top(q)$ and $T_{l(q)}\mathcal{L}$, $\forall q \in \mathcal{M}$. Thus, $\Delta^\perp = \ker(l_*) = T_q\mathcal{G}_{l(q)}$ $\forall q \in \mathcal{M}$, implying that $\Delta^\perp$ is integrable with $\mathcal{G}_{l(q)}$ as its integral manifold. ∎

If a projection pair $(l, \mathcal{L})$ does not exist (e.g., nonintegrable $\Delta^\perp$), the locked system cannot have a $(n-m)$-dim. configuration $l(q)$, since, on any $\mathcal{L}^{n-m}$, the most basic position-velocity kinematics relation is violated (i.e., $dl(q)/dt \neq l_*(v^\top)$ instead of (27)). In fact, Proposition 1 can be used to check the impossibility of the existence of such a locked system configuration $l(q)$, given the coordination map h and the underlying dynamics ∇. Of course, if we regulate the shape system s.t. $h(q) = c$, the locked system will have a well-defined configuration on $\mathcal{H}_c$ (see the footnote [4]).

Converse of Proposition 1 holds only locally, since, even if $\Delta^\perp$ is integrable $\forall q \in \mathcal{M}$, a single map $l : \mathcal{M} \to \mathcal{L}$ satisfying (27) in general exists only locally [24]. Theorem 2 below shows that, if h is designed s.t. its foliation is "parallel" w.r.t. ∇, there exists a projection pair $(l, \mathcal{L})$. We first recall the following notions [15]. Let $\gamma : [0, 1] \to \mathcal{M}$ be a smooth curve on $\mathcal{M}$. A vector field $V \in \mathfrak{X}(\mathcal{M})$ is parallel along γ w.r.t. ∇ if $\nabla_{\dot{\gamma}} V = 0, \forall t \in [0, 1]$. For each $v_o \in T_{\gamma(0)}\mathcal{M}$, there exists a unique parallel vector field $V_o(\gamma(t))$ along γ with $V_o(\gamma(0)) = v_o$. The (linear) parallel transport map, $\mathcal{P}^\nabla_{\gamma(0)\to\gamma(1)} : T_{\gamma(0)}\mathcal{M} \to T_{\gamma(1)}\mathcal{M}$, is then defined by $\mathcal{P}^\nabla_{\gamma(0)\to\gamma(1)}(v_o) := V_o(\gamma(1))$.

Theorem 2: Suppose that $\mathcal{M}$ of (3) is complete and simply-connected, and $\Delta^\top$ is invariant w.r.t. the holonomy group [25]

$$\mathrm{Hol}_q := \left\{ \mathcal{P}^\nabla_{\gamma(0)\to\gamma(1)} | \gamma(t) \in \mathcal{M}, \quad s.t. \quad \gamma(0) = \gamma(1) = q \right\}$$

for all $q \in \mathcal{M}$. Then, a projection pair $(l, \mathcal{L})$ exists.

Proof: Invariance of $\Delta^\top$ w.r.t. Hol_q also implies that of $\Delta^\perp$, since $\mathcal{P}^\nabla_{\gamma(0)\to\gamma(1)}$ maps $T_{\gamma(0)}\mathcal{M}$ to $T_{\gamma(1)}\mathcal{M}$ and preserves the orthogonality (i.e., $\langle\langle e_i, e_j \rangle\rangle = \langle\langle \mathcal{P}^\nabla_{\gamma(0)\to\gamma(1)} e_i, \mathcal{P}^\nabla_{\gamma(0)\to\gamma(1)} e_j \rangle\rangle, \forall e_i, e_i \in T_{\gamma(0)}\mathcal{M}$). Then, following [26, Prop. 5.1,Ch. IV], $\Delta^\perp$ is integrable and has a m-dim. integral manifold $\mathcal{G}_q \forall q \in \mathcal{M}$, which is complete and totally geodesic (i.e., every geodesic of (3) stemming from $\mathcal{G}_q$ stays on it all the time). Similarly, $\mathcal{H}_{h(q)}$ is also complete and totally geodesic.

Let us choose a point $q_o \in \mathcal{M}$ and a smooth curve $z(t)$ joining q_o and a point $q \in \mathcal{M}$ s.t. $z(0) = q_o$ and $z(1) = q$. We can then define the projections of $z(t)$ on $\mathcal{H}_{h(q_o)}$ and on $\mathcal{G}_{q_o}$ s.t.: $z'(t) \in \mathcal{H}_{h(q_o)}$ with $z'(0) = q_o$ and $\dot{z}'(t) = \mathcal{P}^\nabla_{z'(0)\to z'(t)} A(t)$, where $A(t) := \mathcal{P}^\nabla_{z(t)\to z(0)}(\dot{z}(t))^\top$; and $z''(t) \in \mathcal{G}_{q_o}$, with $z''(0) = q_o$ and $\dot{z}''(t) = \mathcal{P}^\nabla_{z''(0)\to z''(t)} B(t)$, where $B(t) := \mathcal{P}^\nabla_{z(t)\to z(0)}(\dot{z}(t))^\perp$. From these, we can also construct two maps, $\pi' : \mathcal{M} \to \mathcal{H}_{h(q_o)}$ and $\pi'' : \mathcal{M} \to \mathcal{G}_{q_o}$ s.t., given $q \in \mathcal{M}$, $\pi'(q) := z'(1)$ and $\pi''(q) := z''(1)$. As shown in [26, pp.187], both $\pi'(q)$ and $\pi''(q)$ depend only on q (i.e. end-point of $z(t)$), not on a particular shape of $z(t)$. Furthermore, with $\Delta^\top$ and $\Delta^\perp$ being invariant w.r.t. Hol_q and $\mathcal{M}$ being complete and simply-connected, $\mathcal{M}$ is isometric to $\mathcal{H}_{h(q_o)} \times \mathcal{G}_{q_o}$, and the combined map $\pi := (\pi', \pi'')$ defines an (bijective) isometry of $\mathcal{M}$ onto $\mathcal{H}_{h(q_o)} \times \mathcal{G}_{q_o}$ (de Rham decomposition [26, Th.6.1, Ch. IV]). Since the construction of $\pi'(q)$ uses only the tangential component $(\dot{z}(t))^\top$, π' also satisfies (27). Thus, $(\pi', \mathcal{H}_{h(q_o)})$ defines a locked system projection pair. ∎

[7] In general, the decoupling control (22) is still needed in this case—see [8, Prop.6, Sec.3.5].

This Theorem 2 is granted if $\mathcal{M}$ is Euclidean and $\mathcal{H}_{h(q)}$ are flat planes. See [4], [6] for some applications of this "flat" decomposition. Simple connectedness of $\mathcal{M}$, although limited (e.g., SO(3) is not), is also assumed in Theorem 2 to use de Rham decomposition [26, Th.6.1, Ch. IV], which makes Theorem 2 a global result.

F. Passivity-Based Control Design Example

We want to achieve the following two control objectives simultaneously *and* separately: 1) $v^\top(t) \to v_l^d(t)$, where $v_l^d(t) \in T_q^\top \mathcal{M}$ is a desired locked velocity at $q(t)$; and 2) $h(q) \to c_d$, where $c_d \in \mathcal{N}$ is a constant desired coordination shape. To manifest utility of the preserved Lagrangian structure and passivity, we design proportional-derivative (PD) control laws for both of these objectives.

Let us write $T_\star$ in (22) s.t. $T_\star = T_\star^\top + T_\star^\perp$, where $T_\star^\top \in T_q^{*\top}\mathcal{M}$ and $T_\star^\perp \in T_q^{*\perp}\mathcal{M}$ are, respectively, the locked and shape system controls. We first design $T_\star^\top$ s.t.

$$T_\star^\top := M\left(\nabla_{\dot{q}} v_l^d\right)^\top - K_l(q)\left(v^\top - v_l^d\right) - F^\top \tag{28}$$

where $K_l(q) : T_q^\top \mathcal{M} \to T_q^{*\top}\mathcal{M}$ is a dissipation field on $\mathcal{M}$ s.t. $\langle K_l(q) v_l, v_1 \rangle \geq a\langle\langle v_l, v_l \rangle\rangle$, $\forall v_l \in T_q^\top \mathcal{M}$ with $a > 0$. Then, the closed-loop locked system dynamics becomes

$$M(\nabla_{\dot{q}} e^\top)^\top + K_l(q) e^\top = 0$$

where $e^\top := v^\top - v_l^d$, and, with $W_l(t) := \langle\langle e^\top, e^\top \rangle\rangle/2$ and (5), we have $dW_l(t)/dt = -\langle K_l e^\top, e^\top \rangle \leq -2aW_l(t)$, implying that $e^\top \to 0$ exponentially.

We also design $T_\star^\perp$ s.t.

$$T_\star^\perp := h^*\left[-K_h\left(h(q)\right) h_* v^\perp - d\varphi_h\left(h(q)\right)\right] - F^\perp \tag{29}$$

where: i) $K_h(c) : T_c\mathcal{N} \to T_c^*\mathcal{N}$ is a dissipation field on $\mathcal{N}$ defined similarly as K_l; ii) $d\varphi_h(c)$ is the one-form of a nonnegative potential $\varphi_h : \mathcal{N} \to \Re$ measuring the distance between $h(q)$ and c_d; and iii) $h^* : T_{h(q)}^*\mathcal{N} \to T_q^*\mathcal{M}$ is the pull-back of h defined s.t.,

$$\langle h^* w_h, v \rangle_{\mathcal{M}} = \langle w_h, h_*(v) \rangle_{\mathcal{N}} \tag{30}$$

for any $v \in T_q\mathcal{M}$ and $w_h \in T_{h(q)}^*\mathcal{N}$, with $h^* w_h \in T_q^{*\perp}\mathcal{M}$ (i.e., $T_\star^\perp \in T_q^{*\perp}\mathcal{M}$), since, from (13), $\langle h^* w_h, v \rangle = \langle w_h, 0 \rangle = 0$, $\forall v \in T_q^\top \mathcal{M}$. Note that the PD action of $T_\star^\perp$ in (29) is designed first on $\mathcal{N}$ and then pulled back to $\mathcal{M}$ by h^*.

From (17) with (22) and (29), we can then write the closed-loop shape system dynamics on $\mathcal{N}$ s.t.

$$M_h(q)\nabla_{\dot{q}}^h h_* v^\perp + K_h\left(h(q)\right) h_* v^\perp + d\varphi_h\left(h(q)\right) = 0 \tag{31}$$

where we use the definition of ∇^h (23) and the fact that $h_* M^{-1} h^* = M_h^{-1}$ (from (30) with (2) and (24)). Define $W_h(t) := \langle\langle v^\perp, v^\perp \rangle\rangle/2 + \varphi_h(h(q))$. Then, using Theorem 1 and $d\varphi_h(h(q))/dt = \langle d\varphi_h, h_* v^\perp \rangle$, we have $dW_h(t)/dt = -\langle K_h h_* v^\perp, h_* v^\perp \rangle \leq 0$; with some more assumptions (e.g., [27]), we can further establish $(h(q), h_* v^\perp) \to (c_d, 0)$—see [8] for more details on this.

Note that the locked and shape systems' Lagrangian structure and passivity, intentionally preserved by the passive decomposition, are crucial for these relatively simple PD-controls (28), (29) to work here. Note also that we achieve simultaneous *and* separate locked-shape control, which is often necessary in many applications (e.g., grasping). For this, we assume full control actuation and full sensing of $(q, \dot{q})$ [e.g., to implement (22) with (28) and (29)]. Notice however that the passive decomposition itself and its properties in Sections III-A–III-E still hold even with control/sensing limitations; control design addressing thereof is a topic for future research (see [4], [23], [28] for results in this direction).

IV. Illustrative Example: Coordinated Rotation of Two Agents in SO(3)

We consider two agents, each evolving on SO(3) with the following dynamics: for the k-th agent ($k = 1, 2$)

$$\frac{dQ_k}{dt} = Q_k[w_k]^\wedge, \quad J_k\frac{dw_k}{dt} = [J_k w_k]^\wedge w_k + \tau_k \tag{32}$$

where $Q_k \in \mathrm{SO}(3)$ is the rotation matrix (i.e., $\mathcal{M} = \mathrm{SO}(3) \times \mathrm{SO}(3)$), $J_k = \mathrm{diag}[J_{k1}, J_{k2}, J_{k3}] \in \Re^{3\times 3}$ is the inertia, $w_k = [w_{k1}, w_{k2}, w_{k3}]^T \in \Re^3$ is the angular rate, $\tau_k \in \Re^3$ is the control, all represented in the body frame, and $[\star]^\wedge : \Re^3 \to so(3)$ is defined s.t., for $a, b \in \Re^3$, $[a]^\wedge b = a \times b$.

To describe the coordination aspect, following [29], we define $h : \mathrm{SO}(3) \times \mathrm{SO}(3) \to \mathrm{SO}(3)$ s.t.

$$h(Q_1, Q_2) := Q_1^T Q_2$$

with $\mathcal{N} = \mathrm{SO}(3)$. Using the property of $[w_k]^\wedge$ [21, pp.123], we can then write $dh/dt = Q_1^T Q_2 [w_2 - Q_2^T Q_1 w_1]^\wedge$, and, since $Q_1^T Q_2$ is nonsingular, we can further show that

$$\Delta^\top = \ker(A_h), \quad \Delta^\perp = \ker\left(\Delta_\top^T J\right)$$

where $A_h := [Q_2^T Q_1, -I_{3\times 3}] \in \Re^{3\times 6}$, $J := \mathrm{diag}[J_1, J_2] \in \Re^{6\times 6}$, and $\Delta_\top \in \Re^{6\times 3}$ identifies $\Delta^\top$ (i.e., columns of $\Delta_\top$ constitute bases of the vector space $\Delta^\top$). Here, note that $\dot{h} = 0$ if $w \in \Delta^\top$. $\Delta^\top$ and $\Delta^\perp$ are also orthogonal with each other w.r.t. the inertia metric J.

We can then write $w := [w_1; w_2]$ and $\tau := [\tau_1; \tau_2] \in \Re^6$ s.t.

$$w = \underbrace{[\Delta_\top \quad \Delta_\perp]}_{=:S \in \Re^{6\times 6}} \begin{pmatrix} w_L \\ w_E \end{pmatrix}, \quad \tau = \underbrace{[\Omega_\top \quad \Omega_\perp]}_{=S^{-T} \in \Re^{6\times 6}} \begin{pmatrix} \tau_L \\ \tau_E \end{pmatrix}$$

where $\star_\top, \star_\perp$ identifies $\star^\top, \star^\perp$ respectively, and, rewriting the dynamics in (32) using this, we can achieve coordinate expressions of (15) s.t.[8]

$$J_L \dot{w}_L + Q_L w_L + Q_{LE} w_E = \tau_L \tag{33}$$

$$J_E \dot{w}_E + Q_E w_E + Q_{EL} w_L = \tau_E \tag{34}$$

where the terms with J_L, Q_L and J_E, Q_E are from $M(\nabla_{\dot{q}} v^\top)^\top$ and $M(\nabla_{\dot{q}} v^\perp)^\perp$, while those with Q_{LE}, Q_{EL} from $M(\nabla_{\dot{q}} v^\perp)^\top, M(\nabla_{\dot{q}} v^\top)^\perp$, respectively. We can also show that $Q_{LE} = -Q_{EL}^T$, and $\dot{J}_L - 2Q_L$ and $\dot{J}_E - 2Q_E$ are skew-symmetric (e.g., [3], [8]), manifesting that the Lagrangian structure and passivity of (32) are preserved in (33), (34) and the locked-shape coupling is passive.

We can also derive the shape dynamics on $\mathcal{N}$ in coordinates as follows. Define τ_E in (34) s.t. $\tau_E := Q_{EL} w_L + \tau_{E\star}$, and denote by $w_h = \sum_{k=1}^3 w_h^k \partial/\partial h_k$ the shape system velocity on $\mathcal{N}$ (i.e., $h_*(v^\perp)$) and by $\tau_h = \sum_{k=1}^3 \tau_h^k dh_k$ a control designed on $\mathcal{N}$, which will be pull-backed via h^* to $\tau_{E\star}$(cf. (29)), where, with an abuse of notations, we denote bases of $T_{h(q)}\mathcal{N}$ and $T_{h(q)}^*\mathcal{N}$ by $\partial/\partial h_k$ and dh_k, respectively. We can then have

$$w_h = H_* w_E, \quad \tau_{E\star} = H^* \tau_h$$

where $w_h = [w_h^1, w_h^2, w_h^3]^T$, $\tau_h = [\tau_h^1, \tau_h^2, \tau_h^3]^T$, and the kj-th components, h_{*kj} and h_{kj}^*, of H_* and $H^* \in \Re^{3\times 3}$ are given by $h_*(\tilde{e}_{j+3}) = \sum_{k=1}^3 h_{*kj}\partial/\partial h_k$ and $h^*(dh_j) = \sum_{k=1}^3 h_{kj}^* \tilde{e}_{k+3}^*$, with $\tilde{e}_j$ and $\tilde{e}_j^*$ ($j = 4$, 5,6) being bases of $\Delta^\perp(q)$ and $\Omega^\perp(q)$, respectively. Here, H_* is invertible (due to the statement after (23)) and also $H_*^T = H^*$ (since $h_{kj}^* = h_{*jk}$ from (30)). Define also the

[8]These expressions (33), (34) can also be obtained by applying $w = \sum_{i=1}^3 w_L^i \tilde{e}_i + \sum_{j=1}^3 w_E^j \tilde{e}_{j+3}$ and $\tau = \sum_{i=1}^3 \tau_L^i \tilde{e}_i^* + \sum_{j=1}^3 \tau_E^j \tilde{e}_{j+3}^*$ to (15) as done for (21), where $\Delta^\top(q) = \mathrm{span}\{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3\}$, $\Delta^\perp(q) = \mathrm{span}\{\tilde{e}_4, \tilde{e}_5, \tilde{e}_6\}$, $\Omega^\top(q) = \mathrm{span}\{\tilde{e}_1^*, \tilde{e}_2^*, \tilde{e}_3^*\}$ and $\Omega^\perp(q) = \mathrm{span}\{\tilde{e}_4^*, \tilde{e}_5^*, \tilde{e}_6^*\}$, with $\langle \tilde{e}_i^*, \tilde{e}_j \rangle = \delta_{ij}$.

shape metric on $\mathcal{N}$ by $J_h^{ij} := \langle\langle \partial/\partial h_i, \partial/\partial h_j \rangle\rangle_{\mathcal{N}}$. Then, from (24) with $J_E^{ij} = \langle\langle \tilde{e}_{i+3}, \tilde{e}_{j+3} \rangle\rangle_{\mathcal{M}}$, we have $J_h = H_*^{-T} J_E H_*^{-1}$. Using these relations with $\dot{w}_E = H_*^{-1}\dot{w}_h - H_*^{-1}\dot{H}_* H_*^{-1} w_h$, we can then rewrite (34) s.t. [9]

$$J_h \dot{w}_h + Q_h w_h = \tau_h \tag{35}$$

where $Q_h := H_*^{-T} Q_E H_*^{-1} - J_h \dot{H}_* H_*^{-1}$ with $\dot{J}_h - 2Q_h$ skew-symmetric. This (35) is the shape system dynamics mapped to $\mathcal{N}$. This also means that (34) (without $Q_{EL} w_L$) is equivalent to (35), thus, we can choose either (34) or (35) to describe the shape dynamics on $\mathcal{N}$.

We also write the controls (28) and (29) in coordinates s.t.

$$\tau_L = Q_{LE} w_E + J_L \dot{w}_L^d + Q_L w_L^d - B_L \left(w_L - w_L^d \right) \tag{36}$$

$$\tau_E = Q_{EL} w_L - H^* \left[B_h w_h + d\varphi_h^T(h) \right] \tag{37}$$

where $w_L^d(t) \in \Re^3$ is the desired locked velocity, $B_L, B_h \in \Re^{3\times 3}$ are the dissipation gains, and $\varphi_h(h) := k(3 - \mathrm{trace}(Q_1^T Q_2))$, $k > 0$, is a potential defined on $\mathcal{N} = \mathrm{SO}(3)$ [29]. Then, we have

$$\frac{d\varphi_h}{dt} = d\varphi_h \cdot H_* w_E = d\varphi \cdot w = d\varphi \cdot \Delta_\perp w_E$$

where the first equality is from (30), the second equality is due to [29, Eq.(4)] with $d\varphi := [d\varphi_1; d\varphi_2] \in \Re^6$ and $d\varphi_i := -k([Q_i^T Q_j - Q_j^T Q_i]^\vee)^T$, $(i,j) = (1,2)$ or (2,1), and the last equality is because φ_h is a function only of h. This then allows us to compute $H^* d\varphi_h^T(h) = \Delta_{perp}^T d\varphi^T$ for (37) with $H^* B_h w_h = H_*^T B_h H_* w_E$.

Using (36), (37), we can then control the shape and locked systems (i.e., attitude coordination; coordinated rotation) simultaneously and separately. Note that, since $\mathcal{M} = \mathrm{SO}(3) \times \mathrm{SO}(3)$, all the previous $\Re^n$-coordinates passive decomposition results [3]–[6] are not applicable here.

V. Conclusion

We reveal the fundamental passive decomposition property of the mechanical system on a manifold $\mathcal{M}$ with a submersion $h : \mathcal{M} \to \mathcal{N}$, which allows us to achieve simultaneous/separate locked-shape control while exploiting the system's (open-loop) Lagrangian dynamics and passivity. A particularly interesting future research topic is how to include nonholonomy, symmetry, under-actuation and partial state sensing. See [4], [6], [23], and [28] for some results along this line.

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